

Demonstration of Gradient and Curl Concept in Functional Analysis

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Abstract: This paper first establishes the concept of "one-point variation" and points out the improper formulation in this respect, and takes the view that the functional is infinitely many variables. In this way, the concept of gradient is more naturally introduced in functional analysis than in the usual literature. Thus, the criterion of Bahhoepr's Euler operator (i.e. gradient operator) is intuitively and concisely expounded, which makes it almost self-evident. It is difficult for students majoring in electronics to study the course "Electromagnetic Field and Microwave Technology". This is because the theory of this course is strong, the concept is abstract, and the applied mathematics knowledge is more. Especially, it is necessary to understand and master the concept of gradient and curl involved in field theory and two important integral formulas, namely Gauss formula and Stokes formula. However, there are few steps in the common textbooks and this part of the content, so this paper makes a more detailed introduction in order to reduce the difficulties in learning. After discussing the concept of gradient in depth, this method is extended to include natural boundary conditions. The concept of curl is also discussed, and the general form of operator decomposition theorem is given. Contrasting with elementary calculus, the concept of Two-type line integral in functional analysis is formed, which eliminates the contingency of existing parameters in solving inverse variational problems. Finally, some assumptions about the field theory of functional analysis are given, and the criterion of general derivation operator is derived.

1. Introduction

Gradient and curl are important concepts in vector analysis, but many college students and even postgraduates can not correctly distinguish the differences and connections among them [1]. Courant points out that the Euler operator is a gradient, and establishes the criterion of the Euler operator, which extends the concept of self-adjoint operator to the nonlinear case [2]. The concept of curl was put forward in 1974. The formulas for decomposition of general nonlinear operators into gradient and curl in $(B-B^*)$ were established, and the criteria were generalized [3]. But all these do not discuss the relation between boundary value and usual calculus. In order to clarify the above relationship, the author notices that the concept of "variation at one point" is inappropriate in ordinary variational studies. In this paper, we use scalar, vector calculation and legend to analyze the above two degrees of important relations [4]. In this paper, the concept of elementary calculus is improved, and its flexible description is adopted to extend the corresponding field theory concept in functional analysis. Compared with the elementary calculus, the Bahhoepr criterion is described intuitively and concisely, which makes it almost self-evident [5]. In the case of non-fixed boundary conditions, the above concepts are generalized and the decomposition formulas are deduced, that is, not only the criteria of Euler operator and derivative operator are deduced, but also the criteria of additional natural boundary conditions are obtained. The discussion is not limited to the operators in $(B - B^*)$, and the concept of the corresponding two-type line integral in functional analysis is illustrated [6].

2. Gradient of scalar field and curl of vector field

The so-called scalar field is that there is a scalar at all points in the space or part of the space, its

value is a function of spatial position, a scalar field can be expressed by a scalar function. For example, the temperature field in meteorology, potential field in electricity and so on. Similarly, the so-called vector field is that there is a vector at all points in space or part of space, its size and direction is a function of space position, a vector field can be expressed by a vector function. For example, the velocity field in fluid mechanics, the electric field and magnetic field in electromagnetism. In order to investigate the spatial distribution and variation of scalar field, the concepts of isosurface, directional derivative and gradient are introduced. A scalar field can be represented by a scalar function. For example, in Cartesian coordinates, scalar F is a single-valued function F_P of point P in the field, which can be expressed as:

$$F_P = (P_A - P_B) A_P \quad (1)$$

A is a continuous differentiable function of coordinate variables. The equation $(P_A - P_B)$ (P is an arbitrary constant) represents a surface formed by scalar points in a scalar field. It is called an isosurface. For example, the geographic contours, isotherms and so on, in the scalar field, through the isosurface and isotherms can visually understand the physical quantity I in the field of the overall distribution, this is a holistic understanding. The operator is a vector in the direction relation, so its operation has the characteristic of vector operation. Generally speaking, we describe the spatial variation of this scalar field with the maximum rate of change, the relationship between the rate of change in other directions (such as P_i) and the rate of change in P_x direction. It can be estimated as follows: set the scalar value of the P point to i . That is, point P is on the isosurface of $(P_A - P_B)$. The adjacent isosurface is A , the intersection of normal P_x and this isosurface is P_x , P_i and this isosurface is P_i . The change rate in normal direction can be recorded as Q . That is:

$$Q_i = C_q A_i \sqrt{\frac{2\Delta P_i}{\rho}} \quad (2)$$

Similarly, the rate of change along the P_s direction is P , called scalar. The directional derivative of P_s along any direction at point q is:

$$P_s - P_A = \frac{\rho}{2C_q^2 A_1^2} Q_1^2 \quad (3)$$

Because P_i is normal, so:

$$P_R = \frac{P(t+1) - P(t)}{P_N} \quad (4)$$

This expression expresses that the directional derivative is equal to the component of the gradient along the P_R direction, P_R is any direction, if we assign it to be the direction of three coordinate axes in a rectangular coordinate system. Then we have to:

$$P(X \leq R) = P \quad (5)$$

A vector differential operator P is defined. It has the properties of both vector and differential in Cartesian coordinates. Then the gradient of P can be written as:

$$P = P(Y = 1) = F(\beta_i X_i) \quad (6)$$

In other coordinate systems, Y can still be remembered as P , but its form is no longer the upper form. It can be seen that a scalar gradient is a vector field, the vector direction is through the normal direction of the isosurface of each point, and the vector value is the spatial change rate in the normal direction.

In this paper, we mainly refer to this function to evaluate the image quality. The corresponding spatial frequency curves and surfaces are shown in Table 1 and Table 2 below.

Table 1 CSF spatial frequency curve

CSF value	Spatial frequency
0.65	36
0.87	15
0.94	18

Table 2 CSF spatial frequency surface

CSF value	Spatial frequency
0.51	26
0.48	32
0.67	15

Some curves are made in the vector field, so that the tangent direction of each point on the curve is consistent with the corresponding vector direction of the point. The density of the line indicates the size of the vector field of the point. Such a curve cluster is called the force line or streamline of the vector field. These force lines can visualize, describe and analyze the distribution and properties of vector fields, such as streamlines in fluid mechanics, power lines in electromagnetics, magnetic lines and so on. For any area element L in vector field, some curves in vector field are given at the corresponding vector of L location. The tangent direction of each point on the curve is consistent with the corresponding vector direction of the point. The density of the line indicates the vector field of the point. Such a curve cluster is called the force line or streamline of the vector field. These force lines can be used to visualize, describe and analyze the distribution and properties of vector fields, such as streamlines in fluid mechanics, power lines in electromagnetics, magnetic lines and so on, for any area element L in vector fields. The physical quantity obtained by multiplying the component E of the vector RI corresponding to the point L in the normal direction of L by L is called the flux U of dt passing through L . That is:

$$U = RI + L \frac{dI}{dt} + E \quad (7)$$

From the previous discussion, we can see that a vector field with flux source can describe the relationship between field and source by flux and divergence. In order to describe the relationship between the field and the source, the concept of circulation and curl must be introduced.

3. Generalization of gradient concept -- Euler operator, derivation operator and boundary operator

A function of H-element has H self-safety quantities, and when a single change occurs, the function of the function changes to a nearest stable i.e. partial differential. Function as a function, can also be regarded as an infinite multivariable function, directly as an infinite multivariable function. If the self-variable function is limited to continuity, it determines the function at the rational point value, so that infinite values directly determine the functional value. When there is only one change in the infinite variable, the differential is taken, that is, the so-called one-point differential at that point, or the one-point variation. This view is a function of infinite variables.

$$H = [h_1, h_2, \dots, h_k] = A^{1/2} E \quad (8)$$

The position of A and E corresponds to the rate of change. Note that h_k is conditional.

Under the condition of fixed boundary value or natural boundary value, for functional dx , if there is a N operator i . Then:

$$\frac{dx_1^{(1)}}{dt} + ax_1^{(1)} = \sum_{i=1}^N b_i x_i^{(1)} \quad (9)$$

In fixed boundary value or natural boundary value. There are:

$$\hat{a} = (B^T B)^{-1} B^T Y_N \quad (10)$$

Corresponding to the concept of potential force in mechanics, Euler operator is also called potential operator, and B is called Lagrange density of Euler operator.

The gradient and curl of operation are shown in Figures 1 and 2.

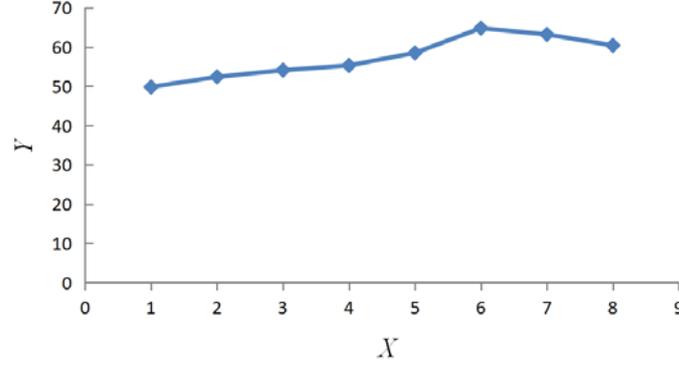


Figure 1 Slice boundary condition

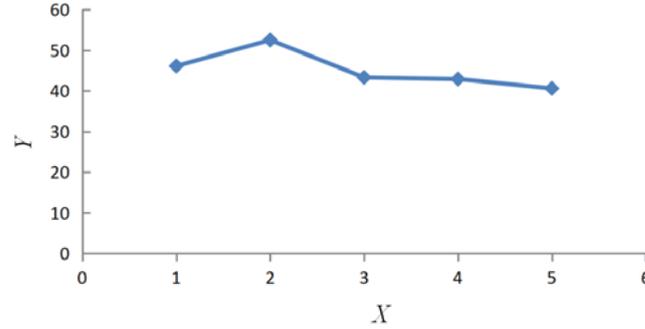


Figure 2 Tangential boundary condition

Under the condition of non fixed boundary and non natural boundary condition. There are:

$$x_1^{(0)}(k) + az_1^{(1)}(k) = \sum_{i=2}^N b_i x_i^{(1)}(k \in K, K = 1, 2, \dots, n, \dots) \quad (11)$$

From the relationship between gradient and differentiation, it can no longer be simply defined. E is gradient. Consider a slightly more complicated example:

$$E_{Rx}(l) = E_{Rx-elec}(l) = lE_{elec} \quad (12)$$

That is, the Lagrange density is W . There are:

$$W = \alpha \left(\beta \left(\frac{E_{i-current}^2}{E_{i-init}^2} \right) + (1 - \beta) \frac{d_i}{d_{max}} \right) \quad (13)$$

In the formula, d is the integral region, d_{max} is its boundary, and α is its outer normal.

$$d_{max} = \max\{d_i\}, i = 1..n \quad (14)$$

When a general functional contains boundary integral:

$$T(n) = \begin{cases} \frac{W * P}{1 - P * [r \bmod(1/P)]}, & n \in G \\ 0, & \text{others} \end{cases} \quad (15)$$

In the formula, T is the boundary operator in the integral of n variation. In the case of no

misunderstanding, it is still remembered as T .

From the above examples, W shows that the change of boundary value (near) changes the rate of change of P , while G means the interior.

For internal operator T and boundary operator n , called (T, n) is a gradient operator, if there is functional E . There are:

$$E_{ch} = lE_{elec} \left(\frac{N}{k} - 1 \right) + lE_{DA} \frac{N}{k} + lE_{elec} + l\xi_{amp} d_{toBS}^4 \quad (16)$$

For simplicity, remember that E is:

$$E_{non-CH} = lE_{elec} + l\xi_{fs} d_{toCH}^2 \quad (17)$$

The relation between inner operator and CH is multiplication, and the relation between boundary operator and lE is approximation by inner operator acting on l . Secondly, the inner part of the gradient operator must be the Euler operator, and the boundary operator which is composed of the same Euler operator can be different. This is because H can still have a non 0 boundary operator when the derivative of X is usually $H=0$.

4. Conclusion

In this paper, the important concepts and physical meanings of gradient and curl are introduced, the important identical relation of two degrees is analyzed, and an example is given to verify it. Combining with the platform of research-based teaching, graduate students can better grasp the second-degree knowledge and significance, strengthen the theoretical basis and numerical programming, and show the role and advantages of research-based teaching. As can be seen from the above discussion, if we consider that the operator is not a normal vector, then we can not talk about the point product and cross product with the vector, and it is not appropriate to use it to represent the gradient and curl. The above gradient operators and curl operators avoid conceptual conflicts. At the same time, as operators are independent, there is no mathematical relationship between them.

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